Discussion sessions of the Research Thematic Trimester on Non-Smooth Complex Systems: January to March 2007

January 15, 15h 30m - Theoretical Analysis: Introductory session

Speakers:
Prof Enrique Ponce, Universidad de Sevilla, Spain
Prof Marco Antonio Teixeira, Campinas State University, Brazil
Prof Slobodan Simic, San Jose State University, USA
Prof Jaume Llibre, Universitat Autònoma de Barcelona, Spain

Abstracts

- S. Simic,
  Structur*al stability of piecewise smooth and hybrid systems*

In the first part of my talk, I will discuss some global and generic aspects of piecewise smooth vector fields, in particular a generic structural stability theorem for Filippov systems on surfaces. This is a natural generalization of M. Peixoto's classical result for smooth vector fields. I will also show that the generic Filippov system can be obtained from a smooth system by a process called pinching. This joint work with M. E. Broucke and C. C. Pugh has precursors in an announcement by V.S. Kozlova about structural stability for the case of planar Filippov systems, and also the papers of J. Sotomayor and J. Llibre and M. A. Teixiera. In the second part of my talk, I will briefly discuss a more general class of hybrid systems, and outline a number of open problems.

- E. Ponce
  Practical Bifurcation Analysis of Piecewise Smooth Systems

The lack of smoothness in these systems precludes the application of local analysis tools (so useful in differentiable dynamics) and then, as a consequence, the separation between local and global issues has a fuzzy character, if it actually exists. Therefore, the bifurcation analysis in this context turns out to be cumbersome and specific for each concrete case. We will show how to proceed in order to obtain bifurcation sets in representative families of PWS systems, and comment the difficulties to overcome in non-solved cases.

- M.A. Teixeira
  Invariant Varities of Relay Systems

The talk will be divided into to main parts:
1- In the first part we recall some facts envolving the connection between a discontinuous vector fields and singularities of mappings. We focus on 2,3 and 4-dimensional systems and classification of generic singularities will be discussed in a local context.
2- We dedicate the second part to present some results concerning Relay Systems in 4D. we focus on the existence of 1-parameter families of closed orbits converging to typical orbits of discontinuous vector fields.

- Jaume Llibre
  Limit cycles in piecewise differential systems via the averaging method.

We present some theorems about averaging theory for piecewise differential systems, mainly for studying their limit cycles and their stability. We do some applications.

January 26, 15h - Theoretical Analysis: Round table discussion.

Participants:

Prof Enrique Ponce, Universidad de Sevilla
Prof Marco Antonio Teixeira, Campinas State University, Brazil
Prof Francisco Torres, Universidad de Sevilla
Prof John. Hogan, University of Bristol, UK.
Prof Mario di Bernardo, University of Bristol, UK.
Prof Tere M-Seara, Universitat Politècnica de Catalunya
Prof Jaume Llibre, Universitat Autònoma de Barcelona

January 29, 15h 30m - Delay equations: Introductory session

Speakers:

Prof Gabor Stepan, Technical University of Budapest, Hungary
Dr Tamas Insperger, Technical University of Budapest, Hungary
Dr Pankaj Wahi, Indian Institute of Science, India.

Abstracts

- G. Stepan
  Time- and state-dependent delays

The linear stability and local nonlinear behaviour of Delayed oscillators are quite well explored in case of constant discrete time delays.
Generally speaking, increasing time delays tend to destabilise equilibria, Hopf bifurcations occur at the stability limits, and these bifurcations tend to become subcritical. These general tendencies are coloured by the opposite effects of time delays especially when the damping terms are small.

In case of state-dependent delays, the numerical and experimental observations show that the the kind of flexibility, or "compliance" of the time delays improve linear stability properties. Linearisation of state-dependent delayed oscillators also supports this observation. However, the local bifurcations in these systems have not been studied yet. Information is available neither analytically, nor numerically. From prcatical view- point, it would be very important to know and understand, whether the improved linear stability properties are accompanied with supercritical Hopf bifurcations at the stability limits, or at least the strength of the subcriticality of the bifurcations decreases. These basic results would provide information to choose the direction of further development in time- delayed systems (like machine tool technology, control of time-delayed systems).

The cartoon problem would be:

$$
\ddot x(t) + ax(t) = f(x(t-\tau(x)))\,,
f(x(t-\tau(x)) = b_1x(t-\tau(x))+b_3x(t-\tau(x)) +\ldots\,,
\tau(x) = \tau_0 + \tau_1 x +\ldots\,.
$$

Time-dependent delayed systems can also improve linear stability properties. On one hand, this improvement is quite sensitive on the proper tuning of the parameters in a complicated way. On the other hand, we do not know much about the effect of time-periodicity on the local nonlinear behaviour. This time-periodicity can show up either in the time delay, or in any other parameter of the system.

It looks obvious that naturally developed neural systems use time-periodic gains in controlling systems with large (and often many) time delays.
While we have already started understanding the advantages of time- periodic control via non-autonomous delay-differential equation models, it is still unexplored how these neural systems work when the time periodicity is established via a kind of nonlinear coupling of a delayed and a non-delayed oscillator like:

$$
\ddot x(t) + ay(t)x(t) = f(x(t-\tau))\,, \ddot y(t) + cy(t) = px(t) + d\dot x(t)\,.
$$

A long-term goal could be to combine the Fitz-Nagumo equations with the delayed oscillator via linear PD control and weak nonlinear coupling.

- T Insperger
  Dynamics of time-periodic and time-delayed systems: milling processes and feedback control systems

Time periodic delayed systems often arise in different fields of engineering. Here, we will concentrate on the dynamics of milling operations and the dynamics of control systems with feedback delay.
The mathematical model of the regenerative vibrations in milling processes is a delayed differential equation with time periodic coefficients. Finding the technological parameters that result in a stable machining process is an important problem for the manufacturing engineers. The corresponding stability properties can be determined using numerical techniques, as it will be shown in some case studies. In control systems, time delays may arise due to the feedback loop or to the digital effects. These delays usually have negative influence on the system's performance. Stabilization or pole placement for delayed systems is not as straightforward as it is for ordinary differential equations. A special case of periodic controllers, the act-and-wait controller is investigated: the controller is switched off for a period just larger than the feedback delay (waiting period) and then it is switched on for a while (acting period).

- P. Wahi
  Self-interrupted turning dynamics: An example of a non-smooth delayed system.

Globally stable solutions for regenerative turning are obtained by incorporating the loss of contact between the tool and the workpiece giving rise to self-interrupted cutting as opposed to parametrically interrupted cutting in milling. The dynamic model of the system switches between infinite-dimensional delay differential equations with different delays and a finite dimensional ordinary differential equation. In this talk, I will outline an alternate approach to model the regenerative effect in metal cutting which leads to a coupled ODE-PDE model with a non-standard non-smooth boundary condition. This approach automatically incorporates the multiple-regenerative effects accompanying self-interrupted cutting. Some lower dimensional ODE approximations are obtained and are used for obtaining a bifurcation diagram of the regenerative turning process. It is found that the unstable branch resulting from the subcritical Hopf bifurcation meets the stable branch resulting from the self-interrupted dynamics in a turning point bifurcation (nature as yet undetermined). A rough analytical estimate of this turning point tool displacement is also obtained. This estimate helps to identify regions in the space of cutting parameters where loss of stability leads to much larger self-interrupted motions than in some other regions. Numerical estimates have also been obtained for global stability, i.e., parameter values for which there exist neither unstable periodic motions nor self-interrupted motions about the stable equilibrium. I would also like to point to some other examples of non-smooth delayed system with rich dynamical behavior which I have recently started working on.

Key issues related to my work:The suitability of the smooth functions used for Galerkin projections and alternate strategies (possibly involving non-smooth functions)for obtaining these projections needs to be discussed during the meeting. Also strategies for determining the types of bifurcations occuring in self-interrupted turning will require to be worked out. These strategies would prove useful in general in dealing with non-smooth systems with delay.

February 9, 15h – Delay equations: Round table discussion.

Participants:

Prof Gabor Stepan, Technical University of Budapest, Hungary

Dr Tamas Insperger, Technical University of Budapest, Hungary
Dr Pankaj Wahi, Indian Institute of Science, India.
Dr David Barton, University of Bristol, UK.
Prof John Hogan, University of Bristol, UK.
Prof Mario di Bernardo, University of Bristol, UK.

February 12, 15h 30m - Bifurcations: Introductory session

Speakers:

Prof Gerard Olivar, Universidad Nacional de Colombia
Dr Michael Schanz, University of Stuttgart, Germany
Dr Viktor Avrutin, University of Stuttgart, Germany
Mr David Barton, University of Bristol, UK.

Abstracts

- G. Olivar
Bifurcations in a Buck DC-DC Power Electronics Converter with different control strategies

Smooth and Nonsmooth bifurcations (also termed Discontinuity-Induced Bifurcations, DIBs) appear in almost any model of a Buck DC-DC Converter, as some parameters are varied. Several classical controllers (ramp voltage, hysteresis) and several modern strategies like ZAD (Zero Average Dynamics) have been applied to the Buck converter and with the variation of some specific parameter, DIBs have been detected. Also, higher codimension bifurcation points, which are dynamics organizers have been identified. In this talk, some examples will be given, which illustrate some DIBs in the Buck converter with different control strategies.

- M Schanz and V Avrutin
How robust is robust chaos?

It is in the meanwhile well-known, that piecewise smooth dynamical systems undergo bifurcations, which can not be observed in smooth systems. However, until now mainly bifurcations involving fixed points and periodic attractors were investigated for piecewise smooth systems. It is also known, that these systems can show chaotic behavior as well, but bifurcations and bifurcation scenarios formed by chaotic attractors are currently still far away from being understood. Especially piecewise smooth systems show often so-called robust chaos (chaos without any periodic windows). The organizing principles of this phenomenon are still not investigated extensively. The short statement "here is chaos" sounds similar to the "here be dragons" inscription on old maps and does not explain, how this chaos emerges and how it is structured. A detailed investigation of these phenomenon in general represents a really challenging task.

In our current work we focus on a particular model, so that our aims are less ambitious than in the general case. Starting from one of the most standard and "most simple" models in the field of piecewise smooth maps, we consider the question, how transitions to chaos take place. When dealing with this question, we detect an infinite number of bifurcation curves organized by a new bifurcation scenario. This scenario explains the structure of the area of chaotic dynamics and is formed by multiband chaotic attractors only, whereby the bandcounts (number of connected components of the attractors) are organized by some kind of infinite adding scheme. This adding scheme is similar to the period adding scheme and hence to the well-known Farey trees. According to this, we detect an infinite number of multiband chaotic attractors with bandcounts increasing to infinity.

Furthermore, we detect, that specific areas organized by this scenario have a complex interior structure, consisting of further areas with increasing bandcounts nested in each other. Currently we are not able to explain this structure in full detail. However, in this talk some hypotheses will be presented, leading us to the assumption, that the area of chaotic dynamics has a self-similar structure. Therefore we conclude, that the area of the so-called robust chaos is in fact robust with respect to its chaotic nature, but is structured by an infinite number of bifurcation curves, leading to an infinite number of different multiband chaotic attractors.

- M Schanz and V Avrutin
  On some generic codimension two and three bifurcation phenomena in non-smooth dynamical systems.

Recently the behavior of piecewise smooth dynamical systems has received significant attention because a large number of systems of practical interest can be modeled by such specific dynamical systems. This includes power electronic circuits, systems involving relays, mechanical systems with impacts and stick-slip oscillations and so on. As shown in several works, the behavior of these systems is strongly influenced by border-collision bifurcations. Remarkably, these bifurcations turned out to be important not only for piecewise smooth dynamical systems, but to be of a more general interest. This is due to the fact that Poincaré return maps of smooth dynamical systems, like for instance the well-known Lorenz system, are often discontinuous and demonstrate these bifurcations as well. Meanwhile codimension one border-collision bifurcations are investigated quite well. However, when investigating codimension one bifurcation scenarios in extended parameter intervals one observes often bifurcation sequences, which are very difficult to understand. Most unexpectedly, increasing the dimension of the considered parameter space, it is possible to obtain much better understandable bifurcation structures, caused by bifurcations of codimension two and three. These discontinuity induced bifurcations serve as organizing centers in multi-dimensional parameter spaces and dominate their structure.

In the presentation, an introduction of those aspects of bifurcation theory will be given which are relevant for the above mentioned topics. It will be shown how the investigation of a dynamical system under variation of one control parameter may lead to results which are very difficult to interpret. These results become well-understandable if one considers the corresponding structures in the two-dimensional and even three-dimensional parameter space, although an infinite number of bifurcation curves are involved. A classification of typical and generic codimension two bifurcations is given and additionally, some codimension three bifurcations will be presented, which are assumed to be of a generic type.

- D Barton
  Numerical continuation for piecewise-smooth delay equations

Numerical continuation in piecewise-smooth (PWS) systems is currently in its infancy; most numerical methods for PWS systems are solely for efficient simulation and these are not always suitable in a continuation context. Here we demonstrate a new numerical method based on collocation with orthogonal polynomials for the continuation of periodic orbits of PWS delay differential equations. This approach enables us to continue both stable and unstable solutions in the system parameters and so build up a picture of the global dynamics. Collocation does not suffer from the numerical stability issues associated with shooting methods and, furthermore, the eigenvalues of the periodic orbit are calculated in a single step without the need for computing saltation matrices. We illustrate this method with the example of chattering behaviour in a model of metal turning.

February 23, 15h - Bifurcations: Round table discussion.

Participants:

Prof Gerard Olivar, Universidad Nacional de Colombia
Dr Michael Schanz, University of Stuttgart, Germany
Dr Viktor Avrutin, University of Stuttgart, Germany
Dr David Chillingworth, University of Southampton, UK
Prof John. Hogan, University of Bristol, UK.
Prof Mario di Bernardo, University of Bristol, UK.
Prof Enric Fossas, Universitat Politècnica de Catalunya

February 26, 15h 30m – Numerics and Applications I: Introductory session

Speakers:

Dr Martin Homer, University of Bristol, UK
Dr Petri Piiroinen, KTH, Sweden and University of Bristol, UK
Dr David Chillingworth, University of Southampton, UK

Abstracts

- D. Chillingworth
  Local and global geometry in the dynamics of an impact oscillator.
  As is well known, in the dynamics of an impact oscillator the so-called grazing orbits play a key role. A geometric model for understanding the geometry of grazing (nondegenerate or otherwise) has been proposed earlier: in this talk we study some local and global dynamical consequences. In particular, we focus on the existence of hyperbolic invariant sets in both low-velocity and high-velocity regimes.

- P Piiroinen
  Future challenges in the analysis in piecewise smooth systems.
  In this presentation I will take the opportunity to highlight possible future directions in which the analysis of piecewise smooth (PWS) systems can go. In particular I will focus on discontinuity induced bifurcations and to what extent they relate to what happens in real- world systems. I will speculate around the following questions. In what situations and for what systems does local analysis reflect what we see in experiments? For instance, for what type systems can we expect local maps to pinpoint what happens in the corresponding real real-world systems? Is there a difference between, for instance, mechanical systems and systems found in electronics in how reliable a local analysis is? Does chattering (an infinite accumulation of events in a finite time) exist in real-world systems? When creating a mathematical model, for what systems is the introduction of a certain (set-valued) discontinuity correct, and when is it appropriate to smooth out a discontinuity?

  I will also discuss the importance of understanding the interaction between grazing boundaries and stable and unstable manifolds from a dynamical system and an experimental point of view.

  Furthermore, I will discuss some future challenges in numerical analysis of PWS systems, e.g. continuation and branch switching, manifold calculation, stochastisity and discontinuity surfaces with corners.

- M Homer (*TBA*)

March 9, 15h - Numerics and Applications I: Round table discussion.

Participants:

Dr Martin Homer, University of Bristol, UK
Dr Petri Piiroinen, KTH, Sweden and University of Bristol, UK
Dr David Chillingworth, University of Southampton, UK
Prof Gerard Olivar, Universidad Nacional de Colombia
Prof John. Hogan, University of Bristol, UK.
Prof Mario di Bernardo, University of Bristol, UK.

March 12, 15h 30m – Numerics, Applications and Control: Introductory session

Speakers:

Prof Alan Champneys, University of Bristol, UK
Prof Harry Dankowicz, University of Illinois at Urbana Champaign, USA

Abstracts

- A Champneys
  Towards a qualitative bifurcation theory for piecewise smooth systems

- H Dankowicz
  On the stabilizability of near-grazing dynamics -- thoughts on the control of (and using) discontinuity-induced bifurcations

In addition to instabilities already present in smooth dynamical systems, vibro-impact oscillators are susceptible to dramatic changes in system response, including the sudden disappearance of a system attractor, that are a direct consequence of their piecewise smooth nature. Control algorithms that aim to suppress such changes in system response must accurately account for the source of the instability. Conversely, the natural dynamics of piecewise-smooth systems may be exploited in control design for purposes of achieving a desirable behavior in originally smooth systems.

In this talk, particular emphasis is placed on the changes in system response that occur as a results of the onset of low-velocity impacts with an obstacle in an originally non-impacting, asymptotically stable, periodic oscillation. Such grazing bifurcations are known to be associated with large amounts of state-space stretching that may result in the sudden disappearance of a local recurrent motion (and, consequently, a local system attractor) or, when combined with a sufficient degree of folding, a robustly-chaotic low-velocity-impacting response. In the former case, the onset of low-velocity impacts typically results in a dramatic and sudden jump in system response to a non-local impacting system attractor with high-velocity impacts.

A constructive proof is presented for the existence of event-driven control strategies that guarantee the local persistence of system attractors with at most low-velocity contact in vibro-impacting oscillators. In particular, sufficient conditions are formulated on the linearization of the control strategies along a grazing periodic trajectory, i.e., an oscillating motion that achieves zero-relative-velocity contact with a mechanical obstacle, to ensure the asymptotic stability of the grazing trajectory and, consequently, sustained dynamics in the vicinity of the grazing trajectory even under small changes in system parameters. The implications of the methodology are illustrated with linear and nonlinear, single- and multiple-degree-of-freedom examples of vibro-impact oscillators.

Finally, some thoughts are presented as to extensions of the theory to Filippov systems and to smooth dynamical systems with regions of rapid variability in the vector field in which a purposeful coarsening to a piecewise-smooth model may be possible.

March 23, 15h - Numerics, Applications and Control: Round table discussion.

Speakers:

Prof Alan Champneys, University of Bristol
Prof Harry Dankowicz, University of Illinois at Urbana Champaign, USA
Dr David Chillingworth, University of Southampton, UK
Prof Gerard Olivar, Universidad Nacional de Colombia
Prof John. Hogan, University of Bristol, UK.
Prof Mario di Bernardo, University of Bristol, UK.
Prof Enric Fossas, Universitat Politècnica de Catalunya